

June 2004

GCE A AND AS LEVEL

MARK SCHEME

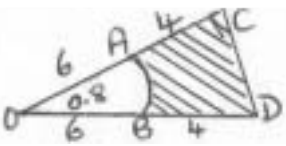
MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/01


MATHEMATICS
Paper 1 (Pure 1)



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<p>1. (i) $a/(1-r) = 256$ and $a = 64$ $\rightarrow r = \frac{3}{4}$</p> <p>(ii) $S_{10} = 64(1-0.75^{10}) / (1-0.75)$ $\rightarrow S_{10} = 242$</p>	<p>M1 A1 [2]</p> <p>M1 A1 [2]</p>	<p>Use of correct formula Correct only</p> <p>Use of correct formula – 0.75^{10} not 0.75^9 Correct only</p>
<p>2. $\int_0^1 \sqrt{3x+1} dx = (3x+1)^{1.5} \div 1.5$</p> <p>then 3</p> <p>$\rightarrow []$ at 1 – $[]$ at 0</p> <p>$\rightarrow 16/9 - 2/9 = 14/9$ or 1.56</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1 [4]</p>	<p>MI for $(3x+1)^{1.5} \div 1.5$</p> <p>For division by 3</p> <p>Must attempt $[]$ at $x=0$ (not assume it is 0) and be using an integrated function</p> <p>Fraction or decimal. (1.56+C loses this A1)</p>
<p>3. (i) $\sin^2 \theta + 3\sin \theta \cos \theta = 4\cos^2 \theta$ divides by $\cos^2 \theta$ $\rightarrow \tan^2 \theta + 3\tan \theta = 4$</p> <p>(ii) Solution $\tan \theta = 1$ or $\tan \theta = -4$ $\rightarrow \theta = 45^\circ$ or 104.0°</p>	<p>M1 A1 [2]</p> <p>M1</p> <p>A1 A1 [3]</p>	<p>Knowing to divide by $\cos^2 \theta$ Correct quadratic (not nec = 0)</p> <p>Correct solution of quadratic = 0</p> <p>Correct only for each one.</p>
<p>4. (i) Coeff of $x^3 = 6C3 \times 2^3$ $= 160$</p> <p>(ii) Term in $x^2 = 6C2 \times 2^2 = 60$ reqd coeff = $1 \times (i) - 3 \times 60$ $\rightarrow -20$</p>	<p>B1 B1 B1 [3]</p> <p>B1</p> <p>M1 A1 [3]</p>	<p>B1 for $6C3$ B1 for 2^3 B1 for 160</p> <p>B1 for 60 (could be given in (i))</p> <p>Needs to consider 2 terms co</p>
<p>5.</p>  <p>(i) Area of sector = $\frac{1}{2} 6^2 0.8$ (14.4) Area of triangle = $\frac{1}{2} 10^2 \sin 0.8$ (35.9) \rightarrow Shaded area = 21.5</p> <p>(ii) Arc length = 6×0.8 (4.8) CD (by cos rule) or $2 \times 10 \sin 0.4$ (7.8) \rightarrow Perimeter = $8 + 4.8 + 7.8 = 20.6$</p>	<p>M1 M1 A1 [3]</p> <p>M1 M1 A1 A1 [4]</p>	<p>Use of $\frac{1}{2}r^2\theta$ with radians Use of $\frac{1}{2}absinC$ or $\frac{1}{2}bh$ with trig Correct only</p> <p>Use of $s=r\theta$ with radians Any correct method – allow if in (i) Correct only</p>

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<p>6. (i) eliminates x (or y) completely $\rightarrow x^2+x-6=0$ or $y^2-17y+66=0$ Solution of quadratic = 0 $\rightarrow (2, 6)$ and $(-3, 11)$</p> <p>(ii) Midpoint = $(-1/2, 8 1/2)$ Gradient of line = -1 Gradient of perpendicular = 1 $\rightarrow y - 8 1/2 = 1(x + 1/2)$ (or $y = x + 9$)</p>	<p>M1 A1 DM1 A1 [4]</p> <p>B1 ✓ M1</p> <p>M1 A1 [4]</p>	<p>Needs x or y removed completely Correct only (no need for = 0) Equation must = 0. Everything ok.</p> <p>For his two points in (i) Use of y-step x-step (beware fortuitous) Use of $m_1m_2 = -1$</p> <p>Any form – needs the M marks.</p>
<p>7. (i) Differentiate $y=18/x \rightarrow -18x^{-2}$ Gradient of tangent = $-1/2$ Gradient of normal = 2 Eqn of normal $y-3 = 2(x-6)$ $(y=2x-9)$ If $y = 0, x = 4 1/2$</p> <p>(ii) $\text{Vol} = \pi \int_{4.5}^6 \frac{324}{x^2} dx = \pi[-324x^{-1}]$ Uses value at $x=6$ – value at $x= 4.5$ $-54\pi - -72\pi = 18\pi$</p>	<p>M1 A1 DM1 DM1 A1 [5]</p> <p>M1 A1 DM1 A1 [4]</p>	<p>Any attempt at differentiation For $-1/2$ Use of $m_1m_2 = -1$ Correct method for eqn of line</p> <p>Ans given – beware fortuitous answers.</p> <p>Use of $\int y^2 dx$ for M. correct(needs π) for A</p> <p>Use of 6 and 4.5</p> <p>Beware fortuitous answers (ans given)</p>
<p>8. (i) $2h + 2r + \pi r = 8$ $\rightarrow h = 4 - r - 1/2 \pi r$</p>  <p>(ii) $A=2rh+1/2\pi r^2 \rightarrow A = r(8-2r-\pi r) + 1/2 \pi r^2$ $\rightarrow A = 8r - 2r^2 - 1/2 \pi r^2$</p> <p>(iii) $dA/dr = 8 - 4r - \pi r$ $= 0$ when $r = 1.12$ (or $8/(4+\pi)$)</p> <p>(iv) $d^2A/dr^2 = -4 - \pi$ This is negative \rightarrow Maximum</p>	<p>M1 A1 [2]</p> <p>M1 A1 M1 A1 DM1 A1 [4]</p> <p>M1 A1 [2]</p>	<p>Reasonable attempt at linking 4 lengths + correct formula for $1/2C$ or C. Co in any form with h subject.</p> <p>Adds rectangle + $1/2$xcircle (eqn on own ok) Co beware fortuitous answers (ans given)</p> <p>Knowing to differentiate + some attempt Setting his dA/dr to 0. Decimal or exact ok.</p> <p>Looks at 2nd differential or other valid complete method. Correct deduction but needs d^2A/dr^2 correct.</p>

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<p>9. $\vec{OA} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \vec{OB} = \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}, \vec{OC} = \begin{pmatrix} 4 \\ 2 \\ p \end{pmatrix}, \vec{OD} = \begin{pmatrix} -1 \\ 0 \\ q \end{pmatrix}$</p> <p>(i) $\vec{AB} = \mathbf{b-a} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ Unit vector = $(2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}) / \sqrt{(2^2+4^2+4^2)}$ = $\pm (2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}) / 6$</p> <p>(ii) $\vec{OA} \cdot \vec{OC} = 4 + 6 - p$ = 0 for 90° $\rightarrow p = 10$</p> <p>(iii) $(-2)^2 + 3^2 + (q+1)^2 = 7^2$ $\rightarrow (q+1)^2 = 36$ or $q^2 + 2q = 35$ $q = 5$ and $q = -7$</p>	<p>M1 M1 A1 [3]</p> <p>M1 DM1 A1 [3]</p> <p>M1 A1</p> <p>DM1 A1 or B1 B1 [4]</p>	<p>Condone notation throughout.</p> <p>Allow column vectors or $\mathbf{i,j,k}$ throughout</p> <p>Use of $\mathbf{b-a}$, rather than $\mathbf{b+a}$ or $\mathbf{a-b}$</p> <p>Dividing by the modulus of "his" \vec{AB}</p> <p>Co (allow – for candidates using $\mathbf{a-b}$)</p> <p>Use of $x_1x_2 + y_1y_2 + z_1z_2$</p> <p>Setting to 0 + attempt to solve co</p> <p>Correct method for length with $\pm\mathbf{d-a}, \mathbf{d+a}$</p> <p>Correct quadratic equation</p> <p>Correct method of solution. Both correct. Or B1 for each if $(q+1)^2=36, q=5$ only.</p>
<p>10. $f: x \mapsto x^2 - 2x, \quad g: x \mapsto 2x + 3$</p> <p>(i) $x^2 - 2x - 15 = 0$ End-points -3 and 5 $\rightarrow x < -3$ and $x > 5$</p> <p>(ii) Uses $dy/dx = 2x - 2 = 0$ or $(x-1)^2 - 1$ Minimum at $x = 1$ or correct form Range of y is $f(x) \geq -1$ No inverse since not 1 : 1 (or equivalent)</p> <p>(iii) $gf(x) = 2(x^2 - 2x) + 3 \quad (2x^2 - 4x + 3)$ $b^2 - 4ac = 16 - 24 = -8 \rightarrow -ve$ \rightarrow No real solutions. [or $gf(x)=0 \rightarrow f(x)=-3/2$. Imposs from (ii)]</p> <p>(iv) $y = 2x + 3$ correct line on diagram Either inverse as mirror image in $y=x$ or $y = g^{-1}(x) = \frac{1}{2}(x-3)$ drawn</p>	<p>M1 A1 A1 [3]</p> <p>M1 A1 A1 [4]</p> <p>M1 M1 A1 [3]</p> <p>B2,1,0 [2]</p>	<p>Equation set to 0 and solved. Correct end-points, however used</p> <p>Co-inequalities – not \leq or \geq</p> <p>Any valid complete method for x value Correct only</p> <p>Correct for his value of "x" – must be \geq</p> <p>Any valid statement.</p> <p>Must be gf not fg – for unsimplified ans.</p> <p>Used on quadratic=0, even if fg used.</p> <p>Must be using gf and correct assumption and statement needed.</p> <p>3 things needed – B1 if one missing. <ul style="list-style-type: none"> • g correct, • g^{-1} correct – not parallel to g • $y=x$ drawn or statement re symmetry </p>
<p>DM1 for quadratic equation. Equation must be set to 0. Formula \rightarrow must be correct and correctly used – allow for numerical errors though in b^2 and $-4ac$. Factors \rightarrow attempt to find 2 brackets. Each bracket then solved to 0.</p>		